Strabismus 0927-3972/03/$ 16.00

Strabismus – 2003, Vol. 11, No. 1, pp. 9–16
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Accepted 5 December 2002

Influence of eye position on stereo matching

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Abstract In animals with binocular depth vision, or stereopsis, the visual fields of the two eyes overlap, shrinking the overall field of view. Eye movements increase the field of view, but they also complicate the first stage of stereopsis: the search for corresponding images on the two retinas. If the eyes were stationary in the head, corresponding images would always lie on retina-fixed bands called epipolar lines. Because the eyes rotate, the epipolar lines move on the retinas. Therefore, the stereoptic system has a choice: it may monitor eye position to keep track of the epipolar lines, or it may give up on tracking epipolar lines and instead search for matches over retina-fixed regions, but in that case the search regions must be 2-D patches, large enough to encompass all possible locations of the epipolar lines in all usual eye positions. We use a new type of random-dot stereogram to show that human stereopsis uses large, retina-fixed search zones. We show that the brain somewhat reduces the size of these search zones by rotating the eyes about their lines of sight in a way that reduces the motion of the epipolar lines. These findings show the link between sensory and motor processes: by considering eye motion we can understand why the brain searches for matching images over 2-D retinal regions rather than along epipolar lines; and by considering retinal correspondence we appreciate why the eyes rotate as they do about their lines of sight.

Key words Binocular vision; stereopsis; visual fields; eye movements; eye position; stereo matching; sensorimotor interaction; computational neuroscience

Introduction Many creatures – usually prey animals – have their eyes on the sides of their heads, giving them a panoramic view of approaching danger. But predators more often have their eyes in the front of the head. In the wedge of space where the visual fields of the two eyes overlap, these animals can achieve binocular depth vision, or stereopsis, which is presumably an advantage when they need to locate and pounce on evasive prey. But this advantage comes at a price: the larger the binocular, stereoptic field, the smaller the overall field of view (see Figure 1). Animals can enlarge their field of vision by moving their eyes or head, and indeed many animals have both stereopsis and mobile eyes. But here we show that eye motion complicates the geometric calculations needed for stereopsis.

And yet they move Information about the depth of a visual object is contained in the relative locations of its images on the retinas of the left and right eyes. So the first step in stereopsis is to identify corresponding images on the two retinas, a process called stereo matching.[1–3] This task can be difficult when there are many similar images present, yet we manage it. With a little practice, for instance, most people can see depth images in random-dot stereograms, which means that we correctly match each of several thousand dots in one eye with its partner in the other eye. How do we do it? One possible way is to use epipolar lines.

The problem to be solved is this: given that an object casts its image onto the retina of one eye, say the left, at a locus L, where is the object in 3-D space? It is impossible to know exactly, based on the image at L alone, but we do know that the object must lie somewhere along the straight line running from L through the optical node of the left eye and out into space. The object may be anywhere on this line; some possible locations are shown in Figure 2 like beads on a wire. Now this line in space projects onto an arc on the other eye's retina, called the *epipolar line* corresponding to L, also shown in Figure 2. So wherever the object may lie along its straight line in space, we know that its image on the right retina must lie somewhere on the epipolar line.

Most theories of stereopsis[4–7] have assumed that the brain is a good enough geometer to know about epipolar lines, so that when it searches for the image corresponding to L, it doesn't bother searching the whole retina of the right eye, but confines its attention to the epipolar line. This strategy would greatly simplify stereo matching, reducing the search from two dimensions to one and ruling out large numbers of false matches. That is why most theories of stereopsis rely on epipolar lines. But most of those theories neglect the fact that the eyes rotate in their sockets.

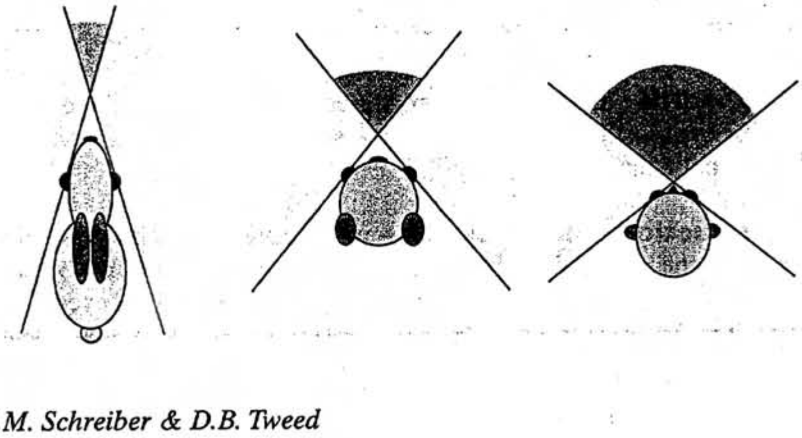


Fig. 1. While lateral-eyed animals have a wide field of view (animal on the left, light and dark grey zones), the binocular portion of the field is small (dark grey). Moving the eyes to the front of the head increases the binocular zone while reducing the overall field of view. The monocular field of view for each eye is the same (180 degrees) in all three animals. Note that the resemblance of the leftmost creature to a rabbit is only superficial, these animals representing a principle rather than actual species.

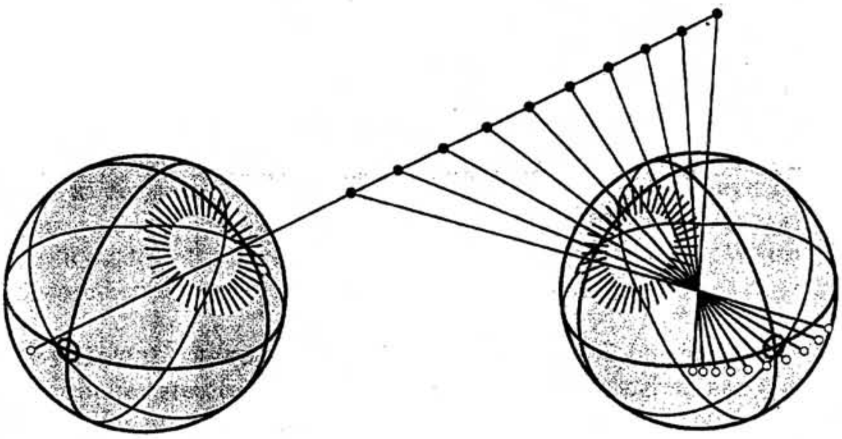


Fig. 2. A visual feature in the left eye's retina (white dot) is the projection of a physical object positioned on a line in space. For several possible positions of that object the projection into the right eye's retina is shown. These possible locations of the corresponding visual feature in the right eye all lie on the epipolar line.

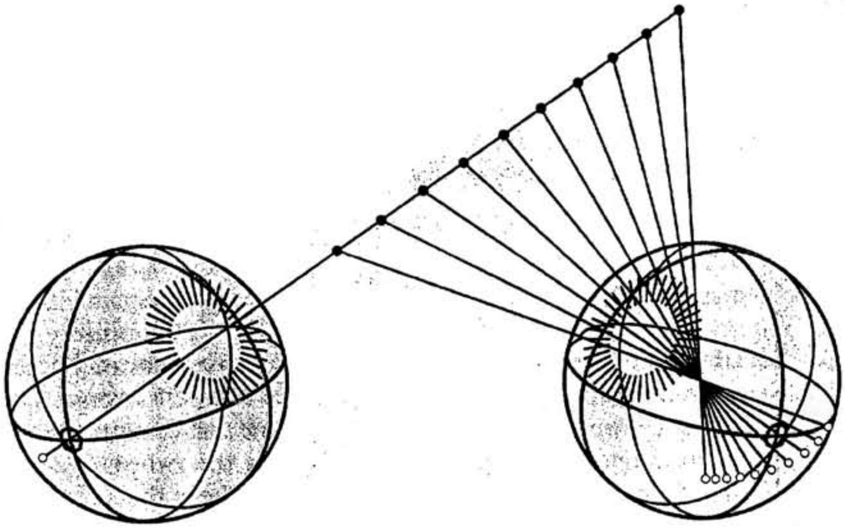


Fig. 3. Here the two eyes have excyclorotated from their positions in Figure 2. The white dot on the left eye's retina is the same as in Figure 2, but its corresponding epipolar line has moved on the right retina.

When either eye rotates, the epipolar lines move on the retinas. In Figure 3 the eyes have rotated a few degrees away from their positions in Figure 2, each eye turning about its line of sight, and now the epipolar line corresponding to the same spot L on the left retina has a new location and orientation on the right retina. This is a major complication for any animal that wants both stereopsis and mobile eyes.

There are essentially two ways that the brain might identify matching images on mobile retinas. One is to monitor current eye position, compute the current locations of the epipolar lines on the retinas, and search for matches only within narrow bands along those lines; we call this method *epipolar matching*. The other way, called *fixed-zone matching*, is to forget about monitoring eye position, and so of course forget about trying to find epipolar lines, and instead search for matches over retina-fixed regions, large enough to cover all possible locations of the epipolar line in any usual eye position. Figure 4 shows nine spots on the left retina, and, superimposed, segments of the corresponding epipolar lines on the right retina. For each spot, the

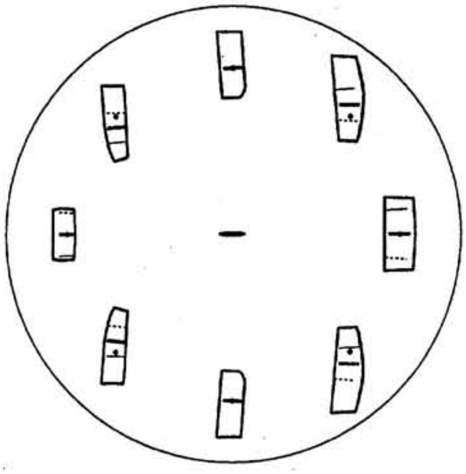


Fig. 4. White dots are nine locations of visual features on the left eye's retina. The corresponding features on the right eye's retina must lie along the thick line when the eyes converge 30 degrees and look level. They lie on the dashed lines when the eyes look up 30 degrees and on the thin lines when they look down 30 degrees, always with 30 degrees of convergence. Outlines mark the entire range of motion of the epipolar segments when the eyes also move 20 degrees right and left. The eyes in this simulation move according to Listing's law, and the retinas are viewed from in front.

figure shows three different epipolar lines, for three different positions of the eyes. If the eyes are allowed to rotate through their entire horizontal and vertical range, these line segments slide around even farther on the right retina, but they remain always within the outlined, 2-D regions marked in the picture, as long as eye rotation is confined to its normal range. These are the retinal regions that must be searched for corresponding images, if the brain uses fixed-zone rather than epipolar matching.

Cyclorotated stereograms Which of the two possible methods of stereo matching does the brain actually use? We can answer this question using cyclorotated stereograms, which are random-dot stereograms, like the one shown in Figure 5, in which the two discs seen by the two eyes are rotated in opposite directions about their centres. Why are these stereograms useful? First of all, they mimic the twisting (or torsional) movements our eyeballs make when we look up or down and converge our eyes on a nearby object in the midsagittal plane of the head. When we converge and look up, our eyes excyclorotate: they twist about their own lines of sight so that the upper poles of the eyeballs rotate outward; when we converge and look down, our eyes incyclorotate.[8–12]

So suppose a person looks up and converges to cross-fuse an excyclorotated stereogram. Suppose that the eyeballs excycloverge 4 degrees, each one twisting 2 degrees outward, and that the stereogram discs are excyclorotated the same way: each disc turned 2 degrees. The twist in the stereogram will cancel the twist of the eyes, so the pattern of correspondence on the two retinas will be just as it would be, if the eyes were looking straight ahead, with no twist at all. So if the brain is adjusting its search based on eye-position information, it should be confused by the twisted stereogram. But if it is searching the same retinal region regardless of eye position, it should actually see the twisted stereogram better. In other words, if the brain is adjusting its search pattern to correct for eye position, then our perception should be unaffected by eye position: the same, untwisted stereograms should be seen best, regardless of whether the eyes are looking up or down. But if the brain is ignoring eye position and instead searching the same retina-fixed zones in all cases, then it should best see stereograms that are rotated with the eyes: excyclorotated stereograms when it looks up, incyclorotated when it looks down. In this case, it should even be possible to devise stereograms that are visible only at certain eye elevations: visible when the eyes look up, for instance, but invisible when they look down, as depicted in the central cartoon in Figure 5. We tested this prediction on five normal subjects.

Positional stereoblindness We presented our subjects with cyclorotated stereograms while they converged 30 degrees and looked 30 degrees up, level, or 30 degrees down. Each stereogram contained one of 20 possible disparity-defined images, which subjects had to identify. We plotted the subject's probability of perceiving the depth image versus the cyclorotation of the stereogram. Figure 6 shows the typical performance of one of our subjects. Data points indicate the fraction of stereograms that this subject could perceive at various cyclorotation angles when the eyeballs were looking level; the curve through the points is a fitted psychometric, or sensitivity, function. For the other two eye positions – 30 degrees up and down – we plot the fitted sensitivity curves without the data points.

Each of the three sensitivity curves has the shape of a mesa, rising steeply from zero on either end to a high plateau in the middle where depth images are perceived with probability 1. But the curves vary with eye position: the whole range of perceptible stereograms shifts toward excyclorotation when gaze is elevated, and toward incyclorotation when gaze is depressed. For instance, a stereogram with 10 degrees of excyclorotation is visible with a probability of almost 1 when the eyes are looking up, and with a probability of just over zero when the eyes are looking down. That is, the stereogram is visible on upgaze but not

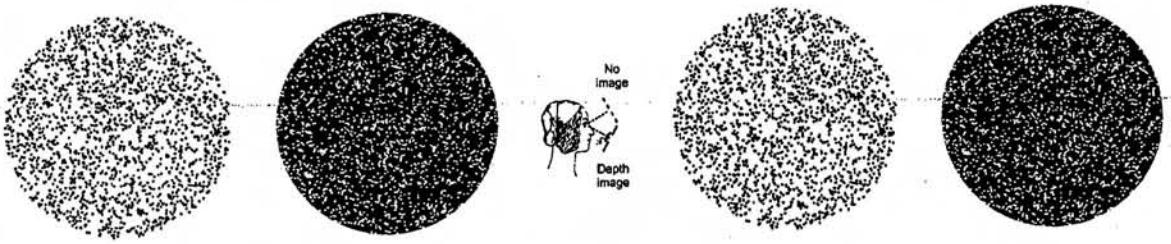


Fig. 5

Fig. 5. Cyclorotated stereograms are visible only in certain eye positions. Cross-fuse the white discs from 20 cm away, depressing your gaze as far as possible and holding the paper orthogonal to the plane of your sight lines (not parallel to your face). You should see a depth image (a triangle pointing downward) in this position, but not when you do the same on upgaze. Flipping the page upside-down yields a stereogram visible only on upgaze. If you can see the shape in up- and in downgaze your search zones are too large; try cross-fusing the black discs, whose cyclorotation angle is greater.

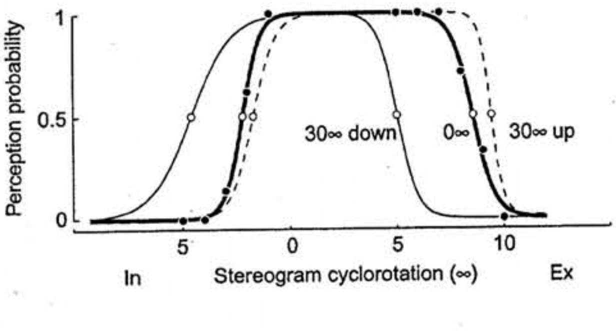


Fig. 6

Fig. 6. Stereopsis depends on gaze elevation. See text for details.

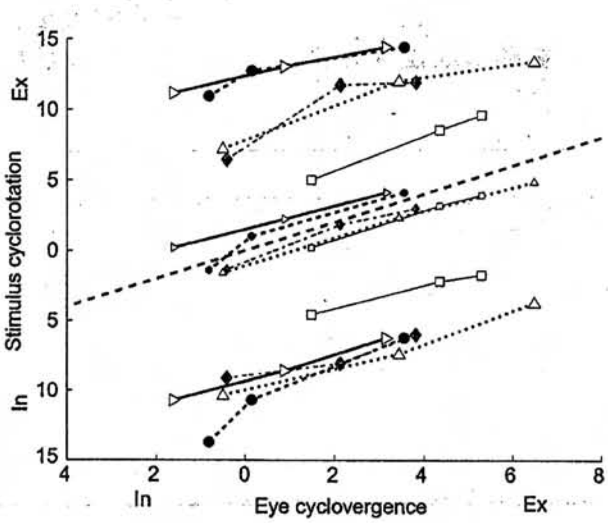
on downgaze. This is the positional stereoblindness which shows that stereopsis works by fixed-zone rather than epipolar matching.

If you are able to free-fuse you can see this for yourself using the cyclorotated stereogram in Figure 5. Hold the page about 20 centimeters from your face and cross-fuse the stereogram. You should see the stereoimage (a triangle) better when you hold the page below eye level, and worse when you hold it above eye level.

Figure 7 shows that in all our subjects, depth perception matched the predictions of the fixed-zone hypothesis. The figure shows, on the ordinate, the angles of stereogram excyclorotation and incyclorotation at which each subject had a 50% chance of perceiving the depth image, plotted versus the cyclovergence of the subject's eyes, measured by a nonius method (subjects monocularly viewed two horizontal lines above and below a binocular fixation point, and rotated the lines until they appeared parallel; the real angle between the lines then reveals the subject's ocular cyclovergence). The subject represented by the diamonds in the figure, for instance, could tolerate 6 degrees of excyclorotation and 9 degrees of incyclorotation when looking 30 degrees down, and at this elevation the subject showed 0.4 degrees of incyclovergence. To summarize each subject's performance in a single curve, the figure shows also the midpoint of the subject's perceptual range, halfway between the thresholds for in- and excyclorotation. For all subjects this curve slopes up to the right; that is, perception depends on eye position, as predicted by the fixed-zone hypothesis.

Quantitatively, too, these data fit the predictions of the fixed-zone hypothesis. If the brain does not use information about eye position to adjust its search at all – if the search zones are absolutely fixed on the retinas – then a one-degree cyclorotation of the eyes should shift the perceptible range of stereograms by one degree. Therefore, the data plotted in Figure 7 should rise with a slope of 1. When we fitted regression lines to the mid-range points of our subjects, the average slope was 1.02, and not significantly different from 1. As accurately as we can measure them, then, the stereo search zones are perfectly fixed on the

Fig. 7. The perception threshold depends on gaze elevation and cyclovergence. See text for details



retinas. The stereoptic system does not try to track the epipolar lines, but instead looks for matches over fixed regions of the retinas.

Oculomotor damage control The search zones must be large enough to encompass all usual positions of the epipolar lines. But the motion of the epipolar lines across the retinas depends in part on the motions of the eyes as they scan the environment. By steering the eyes in a way that minimizes the motion of the epipolar lines, the brain could shrink these search zones, simplifying the computational work of stereo matching. It could do this by rotating the eyes torsionally, about their own lines of sight.

We simulated a wide range of patterns of torsional control, including the one known as Listing's law, which humans use during far vision,[13,14] and another known as L2, or the binocular extension of Listing's law, which we use when we converge on nearby objects.[8–11] For each pattern of ocular torsion we computed the resulting range of motion of the epipolar lines given a normal range of horizontal and vertical eye motion. That range of motion then defined the retinal regions that the brain would have to search in order to ensure that it missed no matches. The results in Figure 8 show that L2 yields a marked advantage over Listing's law, shrinking the required search zones everywhere on the retina. We found that still smaller search zones could be achieved using another pattern of torsion, but the brain has rejected that optimal pattern in favour of L2, which lies about halfway along the continuum between the optimum and Listing's law. This finding suggests that L2 reflects a trade-off between the stereoptic advantages of reduced search zones and the motor efficiency of Listing's law.[12–16]

Sensorimotor interaction Our findings emphasize the close relation between sensory and motor processes. We have shown that stereopsis searches for matches over large, 2-D regions rather than along epipolar lines – a finding that makes sense only when we consider that the eyes move. And we explain human binocular co-ordination – the fact that people abandon Listing's law in favour of L2

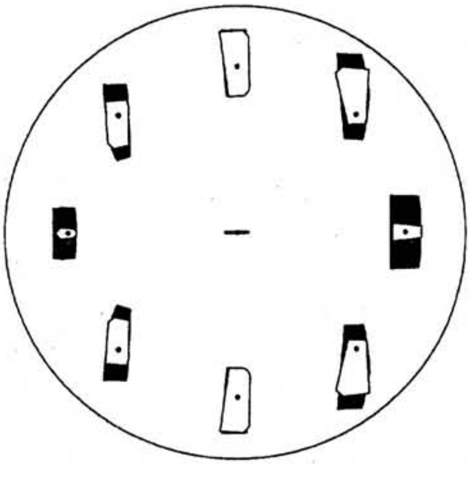


Fig. 8. Stereoptic search zones for Listing's law (black areas) and L2 (white areas). See text for details. The ring of dots is 15 degrees from the fovea, the perimeter circle 20 degrees. Simulated eye positions ranged from −20 to 20 degrees horizontal version, −30 to 30 degrees vertical version and 0 to 30 degrees vergence. The overall area of these search zones for L2 is 63% of that for Listing's law.

when they look up close – by showing that L2 simplifies stereopsis. Neither the sensory nor the motor system can be understood in isolation. And this interdependence is likely to apply quite generally, to other sensorimotor systems besides the stereoptic-oculomotor one. The main task of the brain as a whole is to allow the organism to navigate in and interact with a complex world, perceived through the senses. The entire brain is a sensorimotor system, transforming sensory data into motor output, so understanding it will require a combined sensorimotor approach.

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